**Coursework 3: Control Barrier Functions and Control Lyapunov Functions for Obstacle Avoidance**

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So, the obstacle is defined by:

# Task 2.2

For the first derivative, We use the chain rule:

In simplified form:

Second derivative:

Since , so we plug these in:

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The relative degree of B with respect to control inputs a and ω is:2

**Why Do We Need a Higher-Order Control Barrier Function (HOCBF)?**

Because:

* Standard Control Barrier Functions (CBFs) are only suitable when the relative degree is 1, meaning control inputs appear in the first derivative of the barrier function.
* In our case, the inputs only show up in the second derivative ().

Therefore, to ensure that the system remains in the safe set B(x,y) ≥ 0, and to incorporate the control inputs aaa and ω into the safety condition, we must use a Higher-Order Control Barrier Function (HOCBF).

2.3

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When both V1V\_1V1​ and V2V\_2V2​ approach 0, the robot reaches the goal position (), is oriented toward the goal direction, and moves at the nominal velocity of 1 m/s.

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A distance-only CLF is insufficient because it cannot stabilize both position and orientation.

The chosen V1V\_1V1​ incorporates orientation error and ensures the robot is aligned with the goal direction.